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Physica C 257 (1996) 53-60

PHYSICA C

Is the gap anisotropy in high-T_c superconductors really as high as it is commonly believed?

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Received 2 May 1995; revised manuscript received 15 August 1995

Abstract

Making use of the photoemission data on the dependence of the order parameter Δ on the in-plane momentum $\boldsymbol{k}$ and of the tight-binding fit to the Fermi surfaces in hole-doped high-T_c superconductors, we have calculated the anisotropy parameter χ which enters into the formula for T_c versus impurity concentration in an anisotropic s wave superconductor. We have found that χ is as low as ~ 0.1 in a wide range of hole doping and for different sets of model parameters adjusted to describe the experimental observations of $\Delta(\boldsymbol{k})$ and Fermi surface in $Bi_2Sr_2CaCu_2O_{8+\delta}$. Our results reconcile the large anisotropy of Δ in $\boldsymbol{k}$ space (i.e. not a simple s wave) with the weak sensitivity of T_c to defects and structural inhomogeneities (i.e. probably not a $d_{x^2-y^2}$ wave) and are compatible with the anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$ in hole-doped high-T_c superconductors. Experiments [H. Ding et al., Phys. Rev. B 50 (1994) 1333] on electron irradiation of the twin-free single crystal of $YBa_2Cu_3O_{7-\delta}$ are consistent with a value of $\chi \approx 0.3$.

1. Introduction

The symmetry of the superconducting gap $\Delta(\boldsymbol{k})$ within the CuO_2 planes is the key issue in understanding the pairing mechanism in high-T_c copper-oxide superconductors. While there is clear evidence for the conventional BCS-like pairing in electron-doped high-T_c cuprates [1], experimental results on hole-doped materials still remain controversial (see, e.g., Refs. [2-6]).

Although neither the existence of nodes in the gap function nor the phase change over a $\pi/2$ rotation (which both are characteristic of $d_{x^2-y^2}$ symmetry) have not been definitely established, experimental findings give evidence for a highly anisotropic gap in the a–b planes of hole-doped high-T_c superconductors. This conclusion is based mainly on ultra-high-resolution angle-resolved photoemission spectroscopy studies. For example, Kelley et al. [7] have found that $\Delta(\boldsymbol{k})$ in $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals is as small as 1–2 meV along Γ–Y, 4–8 meV along Γ–X, and 14–20 meV along Γ–M (Cu–O direction in real space). According to Ding et al. [8], $\Delta(\boldsymbol{k})$ in $Bi_2Sr_2CaCu_2O_{8+\delta}$ is minimized, with a value close to zero, along Γ–X and Γ–Y, while $\Delta = 22$ meV along Γ–M. These and many other measurements are consistent with either a $d_{x^2-y^2}$ wave or an anisotropic s wave superconducting state in hole-doped cuprates.

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0921-4534/96/$15.00
SSDI 0921-4534(95)00658-3

To distinguish a $d_{x^2-y^2}$ wave from an anisotropic s wave, one can, in principle, concentrate on differences in physical properties of superconductors with different symmetry of $\Delta(\boldsymbol{k})$: magnetic penetration depth, nuclear magnetic resonance, etc. One possible way, which we adopt in this paper, is to quantify the expected sensitivity of the critical temperature T_c to nonmagnetic impurities for d wave and s wave models [9] and to compare the results with the current experimental situation.

We should note that the approach used in Ref. [9] is a Fermi-liquid-based theory. It is a matter of controversy whether this is really legitimate for copper oxides (see, e.g. Ref. [10]), as strong correlation effects play a key role in these compounds and Fermi liquid picture might not be applicable here. Thus, the standard impurity-scattering theory for superconductors may not be the last word on this issue, but rather a temporary basis for discussion.

2. Nonmagnetic impurities in $d_{x^2-y^2}$ wave superconductors

Anderson's theorem [11] tells us that T_c of an isotropic s wave superconductor is not affected by nonmagnetic impurities, at least if the scattering rate (inverse relaxation time) τ^{-1}, which is proportional to the impurity concentration, does not exceed the Fermi energy E_F (throughout this paper we set $\hbar = k_B = 1$). On the other hand, T_c of a $d_{x^2-y^2}$ wave superconductor is strongly suppressed by a minute concentration of nonmagnetic impurities, a characteristic critical value of τ^{-1} being of the order of T_{c0}, the critical temperature of a clean sample.

The form of the T_c (τ^{-1}) curve for a $d_{x^2-y^2}$ wave superconductor is given implicitly by the relation (see, e.g., Ref. [9])

$$\ln\left(\frac{T_{c0}}{T_c}\right) = \psi\left(\frac{1}{2} + \frac{1}{4\pi\tau T_c}\right) - \psi\left(\frac{1}{2}\right), \qquad (1)$$

where $\psi(x)$ is the digamma function. (Hereafter we restrict ourselves to isotropic impurity scattering.) Note that τ^{-1} in Eq. (1) is the *renormalized* scattering rate, $\tau^{-1} = (1+\lambda)^{-1}\tau_0^{-1}$, where λ is the mass-renormalization constant due to the pairing bosons, and τ_0^{-1} is the bare scattering rate [9]. We stress that the experimentally measured resistivity ρ_0 is determined by the value of τ^{-1}, not τ_0^{-1}, thus facilitating contact with experiment [9]. Moreover, as was shown in Ref. [9], the analytic form of the scaling function T_c/T_{c0} versus $(\tau T_{c0})^{-1}$, Eq. (1), is very close to the computer solutions of the Eliashberg equations for both weak ($\lambda \ll 1$) and strong ($\lambda > 1$) coupling. Hence, Eq. (1) with the renormalized scattering rate τ^{-1} can be used to predict the response of a $d_{x^2-y^2}$ wave superconductor to nonmagnetic impurities.

The initial decrease of T_c is described by the formula

$$\frac{T_c}{T_{c0}} = 1 - \frac{\pi}{8}\frac{1}{\tau T_{c0}} \qquad (2)$$

which follows from Eq. (1) in the limit $(4\pi\tau T_{c0})^{-1} \ll 1$. The superconductivity vanishes ($T_c = 0$) at the critical value $(\tau_c T_{c0})^{-1} = 1.76$. Thus, for the $d_{x^2-y^2}$ wave superconductor with $T_c \approx 100$ K we have $\tau_c^{-1} \approx 20$ meV. Radtke et al. [9] represented the scattering rate τ^{-1} in terms of the planar residual resistivity ρ_0 and predicted that the initial slope $dT_c/d\rho_0$ should be $-(0.8 \div 1.2)$ K/(μΩ cm), and the critical value ρ_{0c} for the destruction of $d_{x^2-y^2}$ wave superconductivity should be $(50 \div 85)$ μΩ cm. (The uncertainties of $dT_c/d\rho_0$ and ρ_{0c} reflect the uncertainty in the experimentally measured plasma frequency ranging from 1.1 to 1.4 eV in $YBa_2Cu_3O_7$).

Recently, Giapintzakis et al. [12] have examined how T_c and $\rho(T)$ change as a function of the electron-irradiation dose and found that $dT_c/d\rho = -(0.30 \pm 0.04)$ K/(μΩ cm), thus ruling out isotropic s wave paring. Though the results of Ref. [12] seem to contradict the prediction of Radtke et al. [9], one should keep in mind that Radtke et al. considered the case of isotropic impurity scattering. As was mentioned in Ref. [12], experimental results are roughly consistent with a d wave superconductor if the impurity scattering has a strong d wave component. At present, however, we are not aware of any systematic experiments on the scattering anisotropy in high-T_c superconductors. So, it is interesting to study in some more detail an alternative way to explain the data of Ref. [12], namely, to consider a suppression of T_c by nonmagnetic impurities in an anisotropic s wave superconductor.

3. Nonmagnetic impurities in anisotropic s wave superconductors

In the case of anisotropic s wave pairing (but isotropic scattering) the functional form of the $T_c(\tau^{-1})$ curve is given by the expression [13,14]

$$\ln\left(\frac{T_{c0}}{T_c}\right) = \chi\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi\tau T_c}\right)-\psi\left(\frac{1}{2}\right)\right], \quad (3)$$

where the coefficient χ is nongeneric. For example, if a circular Fermi surface is considered, $\chi = 1 - 8/\pi^2 \approx 0.19$ for the gap $\Delta(\varphi) = \Delta_0 |\cos(2\varphi)|$ [13], and $\chi = 0.25$ for the gap $\Delta(\varphi) = \Delta_0 |1-(4/\pi)\varphi|$, $0 \leqslant \varphi \leqslant \pi/2$, periodically continued to the interval $[\pi/2, 2\pi]$ [14].

However, a circular Fermi surface is a poor approximation for high-T_c superconductors because of their highly anisotropic electronic structure. In what concerns the dependence of Δ on φ, one can avoid an ambiguity by making use of the photoemission data. A general formula for the parameter χ in the two-dimensional case was given in Ref. [15]:

$$\chi = 1 - \frac{\langle\Delta\rangle^2}{\langle\Delta^2\rangle}, \quad (4)$$

where

$$\langle\ldots\rangle = \frac{\oint(\ldots)\frac{dl}{v(\boldsymbol{n})}}{\oint\frac{dl}{v(\boldsymbol{n})}}; \quad (5)$$

$\boldsymbol{n}$ is a unit vector along the momentum, $v(\boldsymbol{n})$ is the absolute value of the planar quasiparticle velocity, the integration is taken along the Fermi contour (i.e. two-dimensional Fermi surface). Thus, the value of χ is determined by the degree of the order-parameter anisotropy. In particular, $\chi = 0$ if Δ = const. on the Fermi surface (isotropic s wave), and $\chi = 1$ for the $d_{x^2-y^2}$ wave ($\langle\Delta\rangle = 0$). For the anisotropic s wave the parameter χ can take any value between 0 and 1, depending on the actual anisotropy of the superconducting gap.

From a comparison of Eqs. (1) and (3), one can see that T_c of the anisotropic s wave superconductor can be also strongly suppressed by nonmagnetic impurities if the value of the anisotropy parameter χ is large enough. For example, in the case $\chi \sim 1$, the initial sensitivity of the anisotropic s wave to impurities is actually the same as that of the $d_{x^2-y^2}$ wave (the difference between the two consists in the asymptotic approach of T_c to zero for the anisotropic s wave with $\chi \neq 1$ [15], whereas T_c vanishes at some finite $\tau^{-1} \approx T_{c0}$ for the $d_{x^2-y^2}$ wave). For the isotropic s wave ($\chi = 0$) we have T_c = const (Anderson theorem [11]). The initial decrease of T_c at $(4\pi\tau T_{c0})^{-1} \ll 1$ is described by the formula (cf. with Eq. (2))

$$\frac{T_c}{T_{c0}} = 1 - \frac{\pi}{8}\frac{\chi}{\tau T_{c0}}. \quad (6)$$

We emphasize that formulas (3-5) take into account both the anisotropy of the Fermi surface and the in-plane anisotropy of the superconducting gap $\Delta(\boldsymbol{k})$ (or, equivalently, $\Delta(\varphi)$).

At first sight, the strong variation of $|\Delta(\boldsymbol{k})|$ from the Γ–X and Γ–Y to the Γ–M direction, as observed in photoemission experiments [7,8], favors large enough (i.e. close to unity) values of χ, irrespective of the presence or the absence of the sign change of $\Delta(\boldsymbol{k})$ over a $\pi/2$ rotation, i.e. irrespective of $d_{x^2-y^2}$ wave or anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$. Hence, as the condition $T_{c0} \ll E_F$ is obviously fulfilled in high-T_c superconductors, one could expect from Eq. (2) or Eq. (6) a rapid decrease in T_c provided the superconducting state had the $d_{x^2-y^2}$ symmetry or the highly anisotropic s wave symmetry with $\chi \sim 1$.

However, we know that the values of T_c in high-T_c materials do not depend strongly on the sample quality, i.e on the concentration of defects and nonmagnetic impurities. To reconcile this fact with the experimentally observed [7,8] anisotropy of $\Delta(\boldsymbol{k})$, we have calculated the value of χ making use of

(1) photoemission data on the dependence of Δ on $\boldsymbol{k}$ [7], and

(2) the results of the tight-binding fit to the Fermi surfaces in high-T_c superconductors [16].

Some details of our calculations are given below.

We use the form of an order parameter proposed in Ref. [7]:

$$\Delta(\varphi) = \gamma_1\cos(2\varphi) + i\gamma_2\cos(\varphi+\varphi_0), \quad (7)$$

where φ is the angle with respect to the Γ–M direction, and the parameters γ_1, γ_2, and φ_0 are chosen to fit the measured $\Delta(\boldsymbol{k})$ values along the major symmetry directions in the Brillouin zone. According in Ref. [7], the majority of the samples were described by the following parameter values:

$$\gamma_1 = 13.1 \text{ meV}, \gamma_2 = 6.09 \text{ meV, and } \varphi_0 = 118.0°. \quad (8)$$

We would like to stress that the form (7) is nothing more than just the fit to the experimental data on $|\Delta(\varphi)|$, so the presence of terms with $d_{x^2-y^2}$ and $d_{xz}+d_{yz}$ symmetry in Eq. (7) should not be taken too seriously. On an equal footing, the superconducting order parameter in the form (7) may be viewed just as the anisotropic s wave if we deal with the absolute value of Δ. So, assuming the anisotropic s wave order parameter, we set Δ in Eq. (4) to be equal to the absolute value of $\Delta(\varphi)$ given by Eq. (7).

Next, for the description of the Cu–O dpσ antibonding band crossing E_F, we use the simple tight-binding band on a 2D square lattice:

$$\varepsilon(\boldsymbol{k}) = \varepsilon_0 - 2t[\cos(k_x a) + \cos(k_y a)] + 4t' \cos(k_x a) \cos(k_y a), \quad (9)$$

where the parameters ε_0, t, and t' are chosen to fit the Fermi surfaces in hole-doped cuprates [16] (as opposed to parent insulators). The knowledge of the absolute values of the parameters, ε_0, t, and t' is not necessary for our purposes. We need only the relative value t'/t which is equal to 0.45 [16].

Now we proceed with the calculation of the anisotropy parameter χ. First, fixing the number p of doped holes per copper site ($p=0$ at half filling, i.e. $p = 1-n$, where n is the number of electrons per site), we determine the Fermi contour on a 2D lattice. Then we average the values $|\Delta|$ and $|\Delta|^2$ over this contour, $|\Delta|$ being taken in the form (7) with the parameter set (8), and the Fermi velocity $v(\boldsymbol{n})$ in Eq. (5) being simply $[(\partial\varepsilon(\boldsymbol{k})/\partial k_x)^2 + (\partial\varepsilon(\boldsymbol{k})/\partial k_y)^2]^{1/2}$. As a result, from Eq. (4) we obtain the value of χ at a given doping level p.

We have calculated the anisotropy parameter χ in the range $p = 0.1$–0.3, roughly corresponding to the superconducting part of the T_c–p phase diagram of high-temperature superconductors. The most surprising result is that the actual values of χ appeared to be almost an order of magnitude smaller than we have expected from the apparently high anisotropy of Δ in $\boldsymbol{k}+$ space (7). Namely, $\chi = 0.122$–0.126 in the full range of p values studied. It is interesting that the dependence of χ on p is very weak.

To check whether the small χ values arise just from the specific values of the parameters γ_1, γ_2, φ_0 in Eq. (7) and t', t in Eq. (9), we have calculated χ for two other parameter sets in Eq. (7):

$$\gamma_1 = 19.8 \text{ meV}, \gamma_2 = 2.82 \text{ meV, and } \varphi_0 = 0.0°, \quad (10)$$

and

$$\gamma_1 = 12.6 \text{ meV}, \gamma_2 = 16.1 \text{ meV, and } \varphi_0 = 15.3°, \quad (11)$$

as well as for different values of the t'/t ratio in Eq. (9). The set (10) obviously corresponds to a larger anisotropy of $\Delta(\boldsymbol{k})$ as compared with the set (8), since the ratio γ_1/γ_2 for Eq. (10) is much larger than that for Eq. (8). The set (10) is taken from Ref. [7] where it has been proposed as a fit to the photoemission data of Shen et al. [17]. On the other hand, the set (11) clearly points to a smaller degree of gap anisotropy. This set has been found in Ref. [7] for a small fraction of samples studied (though the origin of the parameters variation has not been completely understood in Ref. [7], we would like to stress that a similar effect, i.e. the more isotropic superconducting gap, has been also observed in some samples by Ding et al. [8]).

Our results for the case $t'/t = 0.45$ are as follows: $\chi = 0.178$–0.185 for the set (10), and $\chi = 0.071$–0.072 for the set (11), in accordance with the apparent difference in the gap anisotropy in $\boldsymbol{k}$ space given evidence for by the relative values of γ_1 and γ_2 in Eqs. (8), (10), and (11). In all three cases, the value of χ decreases slightly under hole doping. It is worthwhile to note that the dependence of χ on p weakens with the lowering of the characteristic value of χ in the full range of hole doping studied. We have also calculated the value of χ for various t'/t ratios, and again did not find any appreciable variation.

So, the value of the anisotropy parameter χ is roughly an order of magnitude smaller ($\chi \sim 0.1$) than one could expect from the strong variation of Δ

in $\boldsymbol{k}$ space. But it is the parameter χ that enters into the formulas (3) and (6) for the dependence of T_c on the concentration of nonmagnetic impurities in anisotropic s wave superconductors. Hence, the small values of χ in hole-doped high-T_c superconductors give evidence for the anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$, as they reconcile the strong anisotropy of Δ in $\boldsymbol{k}$ space (i.e. definitely not a simple s wave) with the weak sensitivity of T_c to defects and structural inhomogeneities (i.e. probably neither a $d_{x^2-y^2}$ wave nor an anisotropic s wave with $\chi \sim 1$).

4. Comparison with experiment

It follows from Eqs. (2) and (6) that the initial rate of T_c decrease appears to be an order of magnitude smaller in the anisotropic s wave superconductor with $\chi \sim 0.1$ than in the $d_{x^2-y^2}$ wave superconductor with the same value of T_{c0}. As was shown by Radtke et al. [9], strong-coupling effects have no appreciable influence on the dependence of T_c on τ^{-1} if one uses the renormalized value of τ^{-1} which is proportional to the experimentally measured planar resistivity ρ_0. Following the line of arguments given in Ref. [9], we have converted the $T_c(\tau^{-1})$ dependences into $T_c(\rho_0)$ dependences for different values of the anisotropy parameter χ. The results are presented in Fig. 1 for a 100 K superconductor with the plasma frequency $\omega_{pl} = 1$ eV. This choice of T_{c0} and ω_{pl} is, to some extent, arbitrary. In order to go to the other values of T_{c0} and ω_{pl} one should replace ρ_0 in Fig. 1 by $\rho_0(T_{c0}/100)\omega_{pl}^{-2}$, where T_{c0} is measured in K, and ω_{pl} is measured in eV (see Ref. [9] for details).

It follows from Fig. 1 that $T_{c0} - T_c \sim 1$ K if $\chi \sim 0.1$ and $\rho_0 \sim 20$ μΩ cm. This is in apparent agreement with experimental observations that T_c of high-temperature superconductors do not depend strongly on sample quality, being nearly the same for ceramics, single crystals, and thin films. By contrast, if copper oxides were $d_{x^2-y^2}$ wave superconductors, then T_c would be lowered by ~ 10 K at $\rho_0 \sim 20$ μΩ cm (see Fig. 1), in striking conflict with the experiment. Moreover, the variation of T_c by ~ 1 K from sample to sample is typical for all high-T_c cuprates (including high-quality crystals) and reflects the difficulties in controlling the minute defect concentrations and structural imperfections (on the other hand, T_c's of conventional superconductors with isotropic s wave symmetry of $\Delta(\boldsymbol{k})$ are known with accuracy ~ 0.1 K).

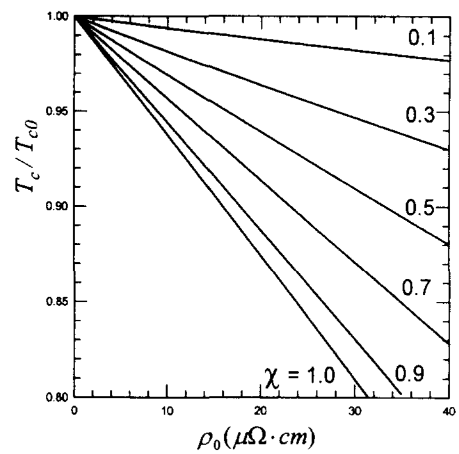

Fig. 1. Normalized critical temperature T_c/T_{c0} as a function of the in-plane residual resistivity ρ_0 due to nonmagnetic impurities in a 100 K superconductor with a d wave ($\chi = 1$) or an anisotropic s wave ($\chi = 0.1, 0.3, 0.5, 0.7, 0.9$) order parameter. The plasma frequency is chosen to be $\omega_{pl} = 1$ eV. One can easily go to the other values of T_{c0} and ω_{pl} through replacing ρ_0 by $\rho_0(T_{c0}/100)\omega_{pl}^{-2}$, where T_{c0} is measured in K, and ω_{pl} is measured in eV [9].

Fig. 2 shows the dependences of T_c/T_{c0} on ρ_0 for 90 K superconductors with $\chi = 1$ and 0.3 along with the experimental dependence [12] of T_c/T_{c0} of $YBa_2Cu_3O_{7-\delta}$ untwinned single srystal on the difference between the values of $\rho(T = 145$ K) in electron-irradiated and -unirradiated samples. The theoretical curves were computed for plasma frequencies $\omega_{pl} = 1.1$ and 1.4 eV. This choice of ω_{pl} reflects the range of experimental uncertainty in ω_{pl} in $YBa_2Cu_3O_{7-\delta}$ (see Refs. [9] and [12]).

From Fig. 2 one can see that experimental data are consistent with the anisotropic s wave superconductivity with the gap anisotropy parameter $\chi = 0.3$. This value of χ is a factor of 2 to 3 larger than that calculated above for the compound $Bi_2Sr_2CaCu_2O_{8+\delta}$. However, it is not clear to what extent the change of $\rho(T = 145$ K) under irradiation reflects the change of ρ_0. As it was mentioned in Ref. [12], the extrapolation of (T) to $T = 0$ gives negative values of ρ_0, so the resistivity in the absence of superconductivity could not remain linear down to

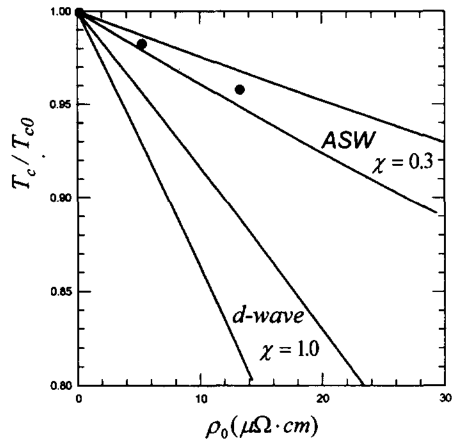

Fig. 2. T_c/T_{c0} vs. ρ_0 in a 90 K superconductor with a d wave or an anisotropic s wave (ASW) order parameter ($\chi = 0.3$) and plasma frequencies ω_{pl} ranging from 1.1 to 1.4 eV (upper and lower curve for a given value of χ). This choice of ω_{pl} reflects the range of experimental uncertainty in ω_{pl} in $YBa_2Cu_3O_7$. The solid points show the experimental data on an electron-irradiated untwinned single crystal of $YBa_2Cu_3O_7$ taken from Ref. [12].

$T = 0$. Thus, the change in $\rho(T = 145$ K) under irradiation may be appreciably different from the corresponding change in ρ_0. Moreover, the choice of $\rho_0 = 0$ in the unirradiated sample adopted in Ref. [12] and used by us in Fig. 2 is probably not correct, as even "high-quality" crystals have more or less impurities and structural imperfections. Taking this fact into account will result in shifting the experimental points of Fig. 2 to the right side, thus leading to a lower value of χ, closer to that in $Bi_2Sr_2CaCu_2O_{8+\delta}$. It would be interesting to conduct experiments on irradiation of high-temperature superconductors with low T_{c0} values, e.g., $Bi_2Sr_2CuO_{6+\delta}$ with properly adjusted oxygen content, in order to make possible a direct determination of the residual resistivity ρ_0.

Besides, the data of Ref. [12] are confined to very low irradiation doses, so that the change in T_c under irradiation is comparable to or less than the width of the resistive transition ΔT_c. This gives rise to an additional error stemming from the uncertainty in $T_{c0} - T_c$. Thus, it would be interesting to broaden the range of $T_{c0} - T_c$ values, keeping the value of ΔT_c roughly unchanged.

In what concerns speculations [12] about a possible strong influence of the scattering anisotropy on the T_c versus ρ_0 curve, we want to note that for a dimensionless constant g_I which enters into the equations for T_c [18], in general, one expects $|g_I| \ll 1$, i.e. very close to the value of $g_I = 0$ for isotropic impurity scattering. On the other hand, it was shown in Ref. [12] that experimental data would be roughly consistent with a d wave superconductor with $g_I = 0.5$. In our opinion, such a large value of g_I (i.e. such a strong d wave component in the impurity scattering) in high-temperature superconductors is unlikely.

5. Discussion and conclusions

Finally, we would like to highlight the physical reason for the small values of the anisotropy parameter χ in hole-doped high-T_c superconductors, which seems to be in a contradiction with the strong anisotropy of Δ in $\boldsymbol{k}$ space. We emphasize that the actual degree of the gap anisotropy is, in fact, determined not only by the dependence of Δ on $\boldsymbol{k}$, but also by the specific form of the Fermi surface. For example, as pointed out by Li et al. [19], in the case of the single-particle dispersion $\varepsilon(\boldsymbol{k}) = -2t[\cos(k_x a) + \cos(k_y a)]$, the extended s wave $\Delta(\boldsymbol{k}) = \Delta_0[\cos(k_x a) + \cos(k_y a)]$ is essentially equivalent to the usual isotropic s wave state $\Delta(\boldsymbol{k}) = \Delta_0 =$ const since the extended s wave state is also constant on the Fermi surface (and we obviously have $\chi = 0$). So, low values of χ in our calculations arose from the weak variation of Δ along the two-dimensional Fermi surface described by Eq. (9), despite the fact that the dependence of Δ on $\boldsymbol{k}$ was rather strong (7). Moreover, the weak influence of the hole-doping level p on the *form* (as opposed to the *area*) of the Fermi surface resulted in a weak dependence of χ on p.

Also, it is interesting to calculate the values of the anisotropy parameter χ for some model dependences $\Delta(\boldsymbol{k})$ or $\Delta(\varphi)$ used in theoretical studies of anisotropic s wave superconductivity. We have computed the values of χ for the cases

(1) $\Delta(\varphi) = \Delta_0|1-(4/\pi)\varphi|$. $0 \le \varphi \le \pi/2$, periodically continued to the interval $\pi/2 \le \varphi \le 2\pi$ [14];
(2) $\Delta(\varphi) = \Delta_0|\cos(2\varphi)|$ [13];
(3) $\Delta(\boldsymbol{k}) = \Delta_0[\cos(k_x a) + \cos(k_y a)]$ [20], making use, as before, of the tight-binding model (5), with $t'/t = 0.45$ and taking the concentration p of doped

holes to be equal to 0.2 (this choice of p roughly corresponds to the maximum of T_c in the T_c–p phase diagram). Our results are as follows:
(1) $\chi = 0.26$.
(2) $\chi = 0.20$.
(3) $\chi = 0.47$.

It is worth noting that in all three cases considered the anisotropy parameter χ varies slightly with p over the interval $0.1 \le p \le 0.3$, as in the case of phenomenological $\Delta(\varphi)$ dependences considered above.

We now discuss the case of extended s wave order parameter $\Delta_{es}(\boldsymbol{k}) = \Delta_0[\cos(k_x a) + \cos(k_y a)]$ more closely. Recently [20] we have studied the influence of site-diagonal Anderson disorder (analogue of non-magnetic impurities) on pairing correlators p_d and p_{es} with d wave and extended s wave symmetries through an exact diagonalization of the Emery hamiltonian on a Cu_4O_8 cluster. We have shown that both p_d and p_{es} decrease with the strength of disorder W (which is proportional to defect concentration), but the effect of disordering on p_d is much more severe than on p_{es}. Pairing correlations in the d wave channel were shown to vanish at a critical value $W_c^d \approx 2t$. However, in Ref. [20] we were unable to determine the corresponding value W_c^{es} in extended s wave channel, though we have noted that W_c^{es} is greater than W_c^d (see Ref. [20] for more details). Now, based on the results obtained in this paper, we may conclude that the initial (i.e. at low defect concentration) sensitivity of the d wave to disordering is twice as high as that of the extended s wave, since $\chi = 1$ and ≈ 0.5 for a d wave and an extended s wave, respectively. Besides, upon an increase of defect concentration the superconducting state with $\chi < 1$ becomes less sensitive to the disorder as compared to the d wave state with $\chi = 1$ [15]. Hence,one would expect that W_c^{es} in cluster calculations exceeds W_c^d more than by a factor of two, i.e. $w_c^{es} > 4t$, in a qualitative correspondence with the results of Ref. [20].

In summary, based on the photoemission data for the dependence of the order parameter Δ on $\boldsymbol{k}$ and on the tight-binding fit to the Fermi surfaces in hole-doped high-T_c superconductors, we have calculated the anisotropy parameter χ which enters into the formula for T_c versus impurity concentration in an anisotropic s wave superconductor. We have found that χ is as low as ~ 0.1 in the wide range of model parameters and hole doping $p = 0.1$–0.3. Our results reconcile the large anisotropy of Δ in $\boldsymbol{k}$ space (i.e. not a simple s wave) with the weak sensitivity of T_c to defects and structural inhomogeneities (i.e. not a $d_{x^2-y^2}$ wave) and are compatible with the anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$ in hole-doped high-T_c superconductors. The experimental data on electron irradiation of an untwinned single crystal of $YBa_2Cu_3O_{7-\delta}$ are consistent with a value of $x \approx 0.3$, though the actual wave of χ may be 0.1–0.2 because of uncertainties in $T_{c0} - T_c$, residual resistivity and plasma frequency.

Acknowledgements

This work was supported by the Russian Scientific Council on High-T_c superconductivity and was performed as a part of Project No. 94031 of the Russian State Program "High-Temperature Superconductivity". The research described in this publication was made possible in part by Grant No. M67000 from the International Science Foundation and by Grant No. M67300 from the International Science Foundation and Russian Government.

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