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Kinetic theory of cascade laser based on quantum wells and wires

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Abstract

The numerical solution of a system of non-linear kinetic equations for electron distribution functions and non-equilibrium phonons is obtained for quantum cascade intersubband lasers on quantum wells and wires. In the case of quantum wells the analytical solution of system of equations is found. The possibility of lowering the threshold current as a result of the accumulation and reabsorption of optical phonons and due to different effective masses of electrons in subbands is demonstrated. For lasers on quantum wires the lowering of threshold current takes place also, with the rate being greater as for lasers on wells.

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Keywords: Quantum cascade lasers; Non-parabolicity; Quantum wires; Accumulation and reabsorption of optical phonons

A new type of semiconductor laser (quantum cascade laser, QCL), operating on electronic transitions between subbands in the conduction band, was demonstrated recently [1]. Because of the absence of a prohibition against intersubband electron transition with optical phonon emission, the high values of the threshold current and the highly non-equilibrium state of electron system are inherent to QCL. Indeed, the electron lifetime in the upper subband (being equal to the time τ_0 of optical phonon emission) is much shorter ($\tau_0 \approx 10^{-12}$–10^{-13} s) as compared to the electron lifetime ($\tau_r \approx 10^{-10}$ s) in the usual semiconductor lasers operating on the transition between valence and conduction bands. That gives rise to the necessity of a kinetic approach to the description of electron energy relaxation in the QCL.

To provide the population inversion, the parameters of the semiconductor hetero-nanostructure were matched in [1] in such a way as to meet the condition $\tau_0 \gg \tau_t$, where τ_t is the tunnelling escape time of electrons from the lower subband. This is essential, if the QCL are considered [1] by analogy with a two-level system ignoring the

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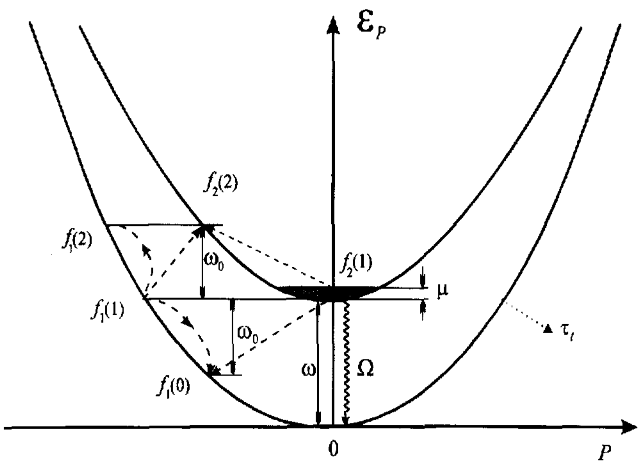

Fig. 1. Dependence of the energy of the subbands on the quasimomentum. The dashed curves correspond to transitions with the emission of an optical phonon, and the wavy curve corresponds to transitions with the emission of a photon.

subbands nonparabolicity. But, as shown in [2], the inclusion of the non-parabolicity changes the situation drastically. In this case the lasing occurs only in the narrow region near the bottom of the conduction band. As a corollary, the stringent condition $\tau_0 \gg \tau_t$ and global population inversion between subbands are not necessary. These facts were confirmed later experimentally [3].

In addition to the importance of subbands nonparabolicity, in [2] one calls attention to the possibility of lowering the threshold current by increasing τ_t ($\tau_t \ll \tau_t$) due to the effects of the accumulation and reabsorption of optical phonons. In fact, these phenomena result to the increase of electron effective lifetime in the upper subband and leads to the lesser injection rate (injection current) required for lasing.

Though the real scheme of the QCL is quite complex, let us consider the following simplified model [2] (see Fig. 1). Let there be two subbands with the dispersion lows $\varepsilon_1(\mathbf{p})$, $\varepsilon_2(\mathbf{p})$, between which transitions occur with the emission of photons having an energy $\hbar\Omega$. Electrons are injected into subband 2 at a rate Q and escape from subband 1 with probability τ_t^{-1} due to the resonant tunnelling. Within the model considered, the main contribution to the scattering processes is made by the interaction of electrons with optical phonons (characteristic energies $\omega_0 \approx 0.034\,\mathrm{eV}$ [3]). The parameter μ ($\mu \ll \omega_0$) corresponds to the width of energetic interval of electrons injected.

The system of kinetic equations for the electron distribution functions $f_1(\varepsilon)$, $f_2(\varepsilon)$ in subbands and non-equilibrium phonons was derived in [2]. As $f_1(\varepsilon)$, $f_2(\varepsilon)$ are localized in narrow energy region $\mu \ll \omega_0$, the functions $f_1(n)$, $f_2(n)$ (where n is an integer number, see Fig. 1) were introduced [2]. The exact analytical solution of this system was found. The injection rate Q was supposed to be small ($Q\tau_0 \ll 1$, linear approximation).

The purpose of the present work is to obtain the threshold injection rate Q_{th} in the wide region of parameters of the model [2]. The solution of the same system of kinetic equations in quasilinear approximation (holding term $Q\tau_0 f_2(1)$) is found. It has the form

$$f_1(n) = A_1 \exp(\alpha n), \quad f_2(n) = A_2 \exp(\alpha n), \quad n \geqslant 2, \tag{1}$$

$$A_1 = Q\tau_0 \frac{(\xi + c)b}{(\xi + 2b)\Delta} \exp(-\alpha), \quad A_2 = Q\tau_0 \frac{(\xi + b)(\xi + c)}{(\xi + 2b)\Delta} \exp(-\alpha), \tag{2}$$

$$f_1(0) = Q\tau_0 \frac{(1 + N)(\xi + c)}{(\xi + 2N)\Delta}, \quad f_1(1) = Q\tau_0 \frac{y}{(\xi + 2N)\Delta},$$

$$f_2(1) = Q\tau_0 \frac{\xi + c - y}{\Delta}, \tag{3}$$

$$\Delta = (\xi + c)(c + Q\tau_0) - (1 + N)(\xi + 2c + Q\tau_0)\left[\exp(\alpha) + \frac{N}{\xi + 2N}\right], \tag{4}$$

$$y = (1 + N)\exp(\alpha) + N(1 + N)/(\xi + 2N), \tag{5}$$

$$b = 2(1 + 2N), \quad c = 1 + 3N, \quad \xi = \tau_0/\tau_t,$$

$$\exp(\alpha) = \frac{b(\xi + b)}{2(1 + N)(\xi + 2b)} - \sqrt{\left[\frac{b(\xi + b)}{2(1 + N)(\xi + 2b)}\right]^2 - \frac{N}{1 + N}}, \tag{6}$$

where N is the number of phonons. N may be found from the equation

$$N\frac{\tau_t}{\tau_{esc}} = f_1(0) - \sum_{n=2}^{\infty}(n - 1)f_1(n), \tag{7}$$

corresponding to the energy conservation low. τ_{esc} is the phonon escape time [2] .

Our main goal is to determine the threshold injection rate. Having obtained the threshold value Δf_{th} of population inversion [2]

$$\Delta f_{th} = \frac{\beta}{\pi\tau\alpha_0}, \qquad \alpha_0 = \frac{e^2|V_{12}|^2}{\Omega\kappa}, \tag{8}$$

where β is the non-parabolicity coefficient [2], τ is the lifetime of photon in the cavity, V_{12} is the matrix element of the transitions between subbands, κ is the dielectric constant, we may find Q_{th} from (1–8), as $\Delta f_{th} = f_2(\varepsilon) - f_1(\varepsilon)$ coincides with $f_2(1)$ at the lasing threshold [2].

Let us consider the limiting situations $\xi \ll 1$ and $\xi \gg 1$. In the first case, performing the expansion in ξ we find after a cumbersome algebra:

$$f_2(1) \approx Q\tau_0 \frac{N}{\xi(1 + N)^2 + Q\tau_0 N}. \tag{9}$$

The characteristic values of N for $Q\tau_{esc} \gg 1$ have proven to be of the order of unity, as follows from (7). Making use of (8) and (9) we obtain

$$Q_{th}(\xi \ll 1) \approx \frac{(1 + N)^2}{N\tau_t}\frac{q}{1 - q}, \qquad q = \frac{\beta}{\pi\tau\alpha_0}. \tag{10}$$

It is seen, that on account of the optical phonons reabsorption, Q_{th} is not defined by τ_0, but by the effective electron lifetime in the upper subband $\tau_t N/(1+N)^2$.

Let us now consider the reverse situation, in which $\xi \gg 1$. Performing the expansion in $1/\xi$ and suggesting, that $\xi \gg N$, we find

$$f_2(1) \approx \frac{Q\tau_0}{Q\tau_0 + N + \sqrt{1+3N+3N^2}}\,. \tag{11}$$

Solving Eq. (7) for N when $Q\tau esc \gg 1$ we estimate the limiting value $N_0 = 1.5$. For the case $\xi \gg 1$ the value Q_{th} may be deduced from (8) and (11):

$$Q_{th}(\xi \gg 1) \approx \frac{5}{\tau_0}\frac{q}{1-q}\,. \tag{12}$$

Comparing the values of Q_{th} from (10) and (12) we obtain

$$\frac{Q_{th}(\xi \ll 1)}{Q_{th}(\xi \gg 1)} \sim \xi\,, \tag{13}$$

where $\xi \ll 1$ in the right part of (13). Therefore, an improvement in the threshold injection rate by a factor of ξ is possible (e.g. by varying the thickness of the barrier at fixed value of τ_0, being restricted by electron–electron and electron–phonon relaxation only.

To verify analytically obtained results, we have solved a system of non-linear equations in wide region of parameters $\tau_{esc}, \tau_0, \tau_t$ and Q numerically by means of iteration method. Details of calculations will be discussed elsewhere [4].

As $f_2(1)$ actually defines the threshold value of population inversion, the behavior of this function seems to be most informative. Figs. 2–4 present typical plots of $f_2(1)$ versus $\tau_t/\tau_0, Q\tau_0$. It is clearly seen from Fig. 2, that the dependence $f_2(1)$ on $Q\tau_0$, calculated with (11) for the limiting case $\xi \gg 1$ coincides with an accuracy of several percents with the numerical solution of system of the kinetic equations [4], and with the solution of that system in the linear and quasi-linear approximation. The same is correct for the case $\xi \ll 1$, but linear approximation is valid up to $Q\tau_0 \sim \xi$.

Fig. 3 demonstrates the increase in $f_2(1)$ with τ_t/τ_0 in agreement with the analytically obtained results (9), (11). So, in the case $Q\tau_0 \ll 1, Q\tau_{esc} \gg 1$ function $f_2(1) \approx 0.027$ for $\tau_t/\tau_0 = 0.1$ and $f_2(1) \approx 0.25$ for $\tau_t/\tau_0 = 10$. It confirms the main idea of the present work on the lowering of threshold current by increasing τ_t.

Up to now the effective masses of electrons in subbands are thought to be the same. The assumption of different masses ($m_2 > m_1$) gives rise to the increase of $f_2(1)$ in comparison with its value calculated for $m_2 = m_1$.

Thus, the numerical solution of the system of non-linear kinetic confirmed the main results of [2] on the kinetic of QCL on quantum wells (QW) and the possibility of threshold injection rate lowering. That solution allows one to specify the area of the validity of linear approximation used in [2]. The linear approximation has proven to be legitimate for $Q < Q_c \sim 1/\tau_t$, i.e., in comparatively narrow region for $\tau_t \gg \tau_0$, and in a much wide region for $\tau_t \ll \tau_0$.

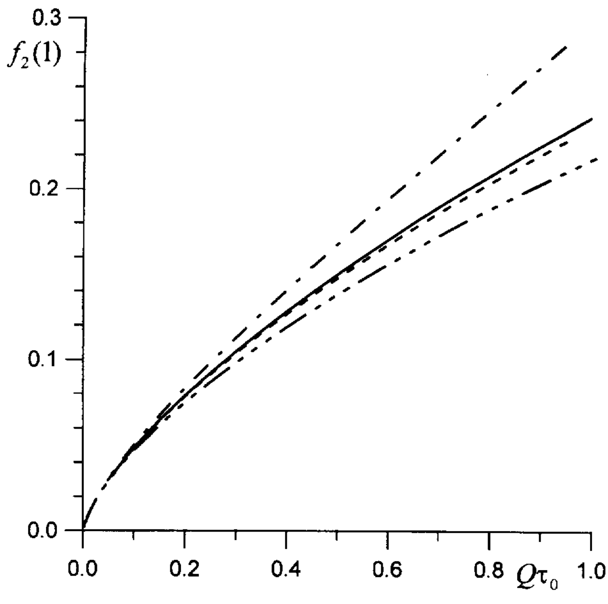

Fig. 2. Dependence of the electron distribution function $f_2(1)$ on $Q\tau_0$. The solid curve is the numerical solution of the system of the kinetic equations, the dashed curve is the solution in quasilinear approximation, the dot-dashed curve is the solution in linear approximation, the dot-dashed curve with 3 dots is calculated with (11). $\tau_{esc}/\tau_0 = 10$. $\tau_t/\tau_0 = 0.1$ ($\xi = 10$).

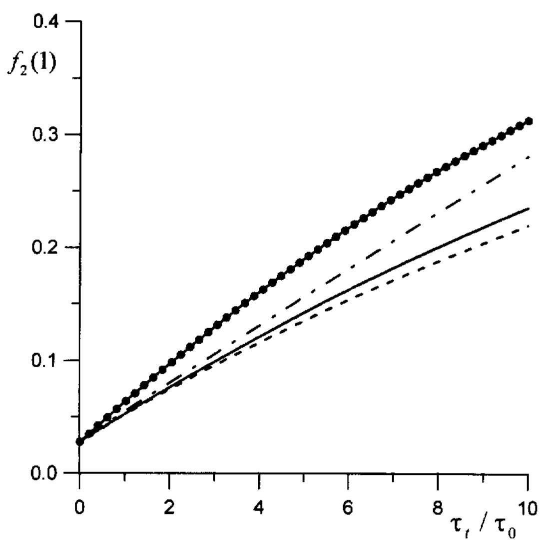

Fig. 3. Dependence of electron distribution function $f_2(1)$ on τ_t/τ_0. The solid curve is the numerical solution of the system of non-linear kinetic equations, the dashed curve is the solution in quasilinear approximation, the dot-dashed curve is the solution in linear approximation. The solid curve with dots corresponds to the case $m_2 = 1.5m_1, \tau_{esc}/\tau_0 = 100$. $Q\tau_0 = 0.1$.

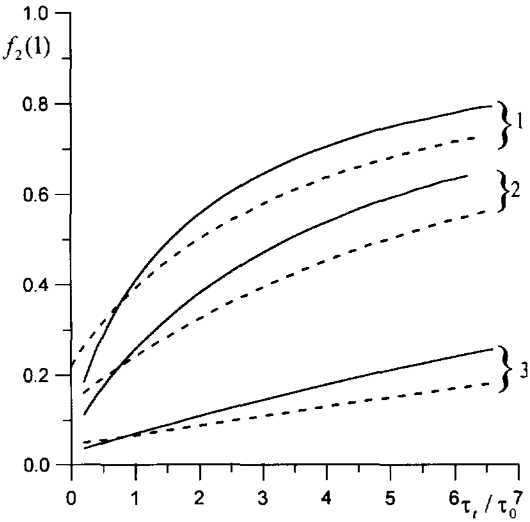

Fig. 4. Dependence of $f_2(1)$ on τ_t/τ_0 for quantum wires (solid curves) and for quantum wells (dashed curves). Curves (1) $Q\tau_0 = 1$,(2) $Q\tau_0 = 0.5$,(3) $Q\tau_0 = 0.1$. $\tau_{esc}/\tau_0 = 10$.

It is interesting also to consider the QCL on quantum wires (QWI). Taking into account the dependence of electronic density of states on energy, the corresponding system of non-linear kinetic equations was derived and solved numerically. Fig. 4 gives the dependence of $f_2(1)$ on τ_t/τ_0 for different values of $Q\tau_0$. For comparison the analogous curves for QCL on QW are shown (dashed curves). It can be seen that in 1D the increase of $f_2(1)$ with τ_t/τ_0 takes place, with the rate being greater as for 2D. So the main features of electron kinetic in two-dimensional QCL on QW are retained for one-dimensional QCL on QWI. The numerical solution of system of kinetic equations for 1D has shown the potential to lower the threshold current by increasing τ_t in QCL on QWI to a greater extent as in QCL on QW. Besides, due to the abrupt decrease of interband electron–electron relaxation rate in the one-dimensional case (see e.g. [5]), the enhancement of τ_t is possible up to large values (10^{-10} s), which provides an improvement in threshold current up to 2–3 orders of magnitude.

The additional decrease of threshold current stems from the singularities in the density of electronic states of one-dimensional system and from different effective masses of electrons in subbands. Thus, one may expect, that QCL on QWI, operating on the transitions between subbands in the conduction band will have the threshold current comparable with currents in the usual semiconductor lasers, working on transitions "band-band".

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